Midterm examination
Business mathematics
$20^{th}$ December 2005
Time: 3 hours (1–4pm)

**1.**
  a. Given
$$f(x) = ax^2 + bx + c$$
  where $a = c = 1$ and $b = 2$. Find all the $x$-intercepts, $y$-intercepts, and critical points.

  b. Given
$$\ln y = 1 - x$$
  Find $y$.

  c. Given
$$\left( \frac{a^x a^y}{a^z} \right)^n = a^b$$
  Find $b$.

  d. Given
$$\log y = 3 \log a + \log b - \log c$$
  Find $y$.

$\qquad$ [10]

**2.**
  a.
$$q = \ln x + \ln y$$
  b.
$$z = x^3 + x^2 + x + 2xy + xy^2$$
  c.
$$p = 150e^{0.74t}$$

Find the first- and second-order partial derivatives.

$\qquad$ [10]

**3.** Find the determinant of the following matrices.
  a.
$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$\qquad$ [2]

  b.
$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$\qquad$ [3]

  c.
$$\begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix}$$

$\qquad$ [5]

**4.** The relationship between the total revenue $r_t$, the price $p$, and the output quantity $q$ is

$$r_t = pq$$

The demand function is $p = a - bq$, where $a$ and $b$ are positive constants.
Find $r_t$, the marginal revenue $r_m$, and the average revenue $r_a$. Then find $q$ at the maximum $r_t$.
And then sketch the graphs of $r_t$, $r_m$ and $r_a$. (assume $a_2 > 4b$)

[10]

**5.**

a. Let $A$ (+ +), $B$ (o o), and $C$ (- -) represent the second-order conditions of critical point of function. Suppose the graph of a function has the shape as shown in Figure 1.
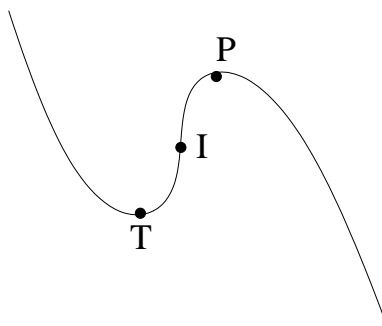


**Figure 1**

Which of these conditions is satisfied at points $I$, $P$ and $T$?[3] Explain.[2]

b.

$$f(x) = \frac{3}{5}x^5 - \frac{9}{4}x^4 + x^3 + \frac{9}{2}x^2 - 6x + 7$$

Find $f'(x)$ and $f''(x)$.[1] Show that $1$, $-1$ and $2$ are the critical points.[2] Which of these are maximum, minimum or inflection point?[2]

**6.** Solve the following programme by the simplex method.

$$\begin{aligned}
\text{maximise:} \quad & z = 3x_1 + 4x_2 + 5x_3 \\
\text{subject to:} \quad & x_1 + x_2 + x_3 \le 2 \\
& x_1 + x_2 + 3x_3 \le 1 \\
& 3x_1 + 2x_2 + x_3 \le 4 \\
\text{with:} \quad & \text{all the variables non-negative}
\end{aligned}$$

[10]